

Electrical Measurement Standards Based on Quantum Phenomena

Dennis Lee
Standards and Calibration Laboratory
Innovation and Technology Commission

Abstract

Worldwide uniformity of measurements is important for breaking down technical barriers to trade. In 1875, the Metre Convention, the second oldest intergovernmental treaty, was established to promote and ensure uniformity of measurements and in 1960, the International System of Units (SI) was established. The equivalence of national measurement standards and hence worldwide uniformity of measurements is ensured through traceability to the SI. The lack of confidence in the measurement standards of an economy would jeopardise the results of tests conducted on its products destined for overseas markets.

The SI base unit for electrical measurements is the ampere for electric current. The realization to high accuracy of the ampere directly in terms of its SI definition is difficult and time consuming. In practice, the ampere for electric current (I) is reproduced indirectly, using Ohm's Law ($I = E/R$), from the volt for electric voltage (E) and the ohm for electric resistance (R). The volt and the ohm can be reproduced with very high accuracy by means of two quantum effects, namely Josephson Effect and Quantum Hall Effect, under specified conditions. This paper gives the metrological background and scientific basis of the primary standards of voltage and resistance based on quantum phenomena at the Standards and Calibration Laboratory (SCL).

1. Why Measurements?

Measurements have always been essential in supporting international trade. In fact, many millions of measurements are made worldwide each day. Ideally, all should be traceable to internationally recognized measurement standards so as to ensure international uniformity. The lack of confidence in the measurement standards of an economy would jeopardise the results of measurements and tests conducted on its products destined for overseas markets. Furthermore, economic success depends upon the ability to manufacture and trade precisely measured and tested products and services. Therefore, worldwide uniformity of measurements is important for breaking down technical barriers to trade.

2. The Global Measurement Infrastructure

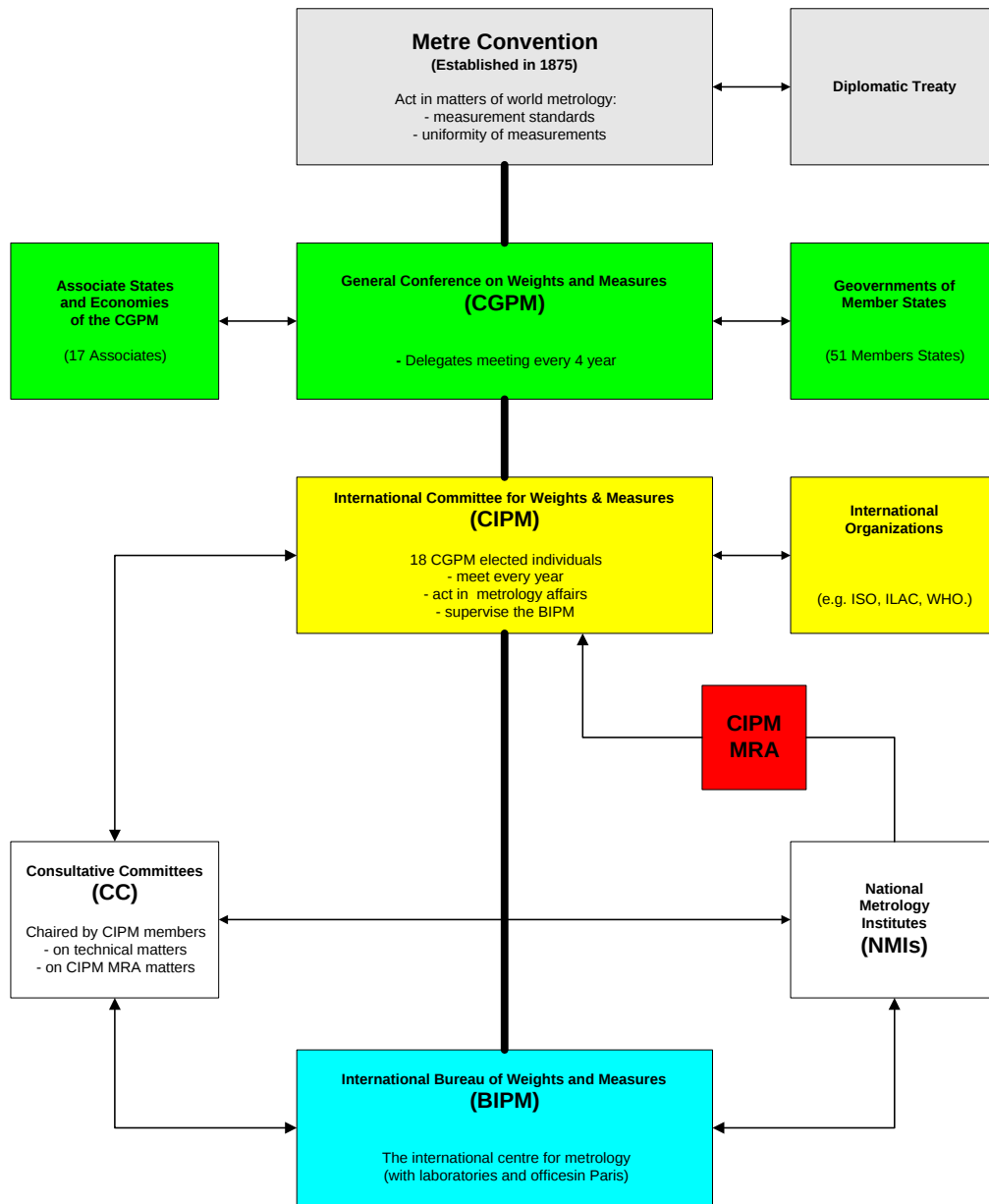


Figure 1: Organization Structure of the Metre Convention

3. The Metre Convention

In 1875, the need for international uniformity in measurements was recognized and formalized by the Metre Convention, which established a permanent global infrastructure for member governments to act in common accord on precise, accurate and traceable measurements. The Metre Convention is the second oldest intergovernmental treaty that gives authority to the General Conference on Weights and Measures (*Conférence Générale des Poids et Mesures*, CGPM) to act in matters of world metrology, particularly concerning the demand for measurement standards of ever increasing accuracy, range and diversity, and the

need to demonstrate equivalence between national measurement standards. The Metre Convention also established the International Committee for Weights and Measures (*Comité International des Poids et Mesures*, CIPM) and the International Bureau of Weights and Measures (*Bureau International des Poids et Mesures*, BIPM).

4. The General Conference on Weights and Measures, CGPM

Today there are 51 Member States of the Convention and 17 Associate States and Economies (non-voting observers) of the CGPM, including Hong Kong, China as represented by the Standards and Calibration Laboratory (SCL), which became an Associate of the CGPM in April 2000. CGPM meets once every four years to discuss and to initiate measures necessary for propagation and improvement of the metric system, and to sanction new fundamental metrological measurements and initiatives, which may have been made between its meetings. It receives a report from the CIPM on the work accomplished and proceeds, by secret ballot, to the renewal of half of the CIPM membership.

5. The International Committee for Weights and Measures, CIPM

The CIPM is made up of members from Member States of the Metre Convention. Its principal task is to promote world-wide uniformity in units of measurement and it does this by direct action or by submitting draft resolutions to the CGPM. CIPM has established ten Consultative Committees, which bring together the world's experts as advisers on scientific and technical matters in the fields of length; mass and related quantities; time and frequency; electricity and magnetism; thermometry; photometry and radiometry; ionizing radiation; amount of substance (metrology in chemistry); acoustic, ultrasound and vibration; and measurement units. The CIPM meets annually and discusses reports presented to it by its Consultative Committees. Tasks of the CIPM include supervising the BIPM; discussing metrological work that Member States decide to do in common; setting up and coordinating activities between specialists in metrology; and making appropriate recommendations to the CGPM.

6. The International Bureau of Weights and Measures, BIPM

The BIPM operates under the exclusive supervision of the CIPM, which itself comes under the authority of the CGPM. The mandate of BIPM is to provide the basis for a single, coherent system of measurements throughout the world, traceable to the International System of Units (*Le Système International d'Unités*, SI). This task of ensuring world-wide uniformity of measurements with SI traceability takes many forms, from direct dissemination of units (e.g. mass and time) to coordination through international comparisons of national

measurement standards (e.g. length, electricity and ionizing radiation).

7. The International System of Units, SI

In 1960, at the 11th CGPM, the SI was established as the metric system of measurement. In 1964, the CIPM set up the Consultative Committee for Units (CCU) to deal with matters concerning units of measurement and the development of the SI. In 1971, at the 14th CGPM, the current version of the SI was completed. The base units are the seven dimensionally independent units, namely the metre (m) for length, kilogram (kg) for mass, second (s) for time, ampere (A) for electric current, kelvin (K) for thermodynamic temperature, candela (cd) for luminous intensity and mole (mol) for amount of substance. Derived units are those formed by combining base units according to the algebraic relations linking the corresponding quantities. The SI is not static but evolves to match the world's increasing requirements for measurement.

8. Worldwide Uniformity of Measurements Through Traceability to the SI

Worldwide uniformity of measurements is essential to eliminate or reduce technical barriers to trade. To achieve this, National Metrology Institutes (NMIs), as designated by the appropriate governmental or other official authority, are tasked with the provision of domestic measurement infrastructures, including measurement standards and calibration services in each economy. The measurement standards of different NMIs but with traceability to the SI have to be compared with each other to ensure equivalence and establish their degree of equivalence. SI traceability means that the value of a standard or the result of a measurement can be related to the SI and, hence, measurements made are nationally and internationally consistent.

9. The CIPM Mutual Recognition Arrangement

In October 1999, a Mutual Recognition Arrangement (CIPM MRA) was signed by the directors of NMIs of 38 Member States of the Metre Convention and representatives of 2 international organizations, namely the International Atomic Energy Agency and the Institute for Reference Materials and Measurements. The CIPM MRA is a response to the need for an open and comprehensive scheme to give reliable quantitative information on national metrology services. The objectives of the CIPM MRA are to establish the degree of equivalence of national measurement standards maintained by NMIs and provide the mutual recognition of calibration and measurement certificates issued by NMIs. On 31 May 2000, the Standards and Calibration Laboratory (SCL), as the NMI of Hong Kong, China became a signatory to the CIPM MRA. As at January 2006, the CIPM MRA has been signed by the

representatives of 67 NMIs and 2 international organizations, covering 112 institutes designated by the signatory bodies. The CIPM MRA provides the technical basis for wider agreements negotiated for international trade, commerce and regulatory affairs.

10. The SI Base Unit for Electrical Quantities: ampere (A)

For electrical measurements, worldwide uniformity and long-term stability of the realization of electrical units are of great importance for science, industry and commerce for both technical and economic reasons. Hence, NMIs must operate within the relevant CIPM recommendations for practical realization of the SI definitions of electrical measurement units. In 1948, the 9th CGPM, adopted the ampere, for electric current, as the SI base unit for electrical measurements, with definition as follows:

The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length.

However, the realization to high accuracy of the ampere directly in terms of its SI definition is difficult and time consuming. Alternatively, the ampere may be indirectly realized through Ohm's Law by realization of the unit of volt and ohm. Thus, the unit of ampere can be related to the fundamental physical constants, namely the Planck constant, h , and elementary charge, e , with recommended values of $6.626\,0693 \times 10^{-34} \text{ J s} \pm 0.000\,0011 \times 10^{-34} \text{ J s}$ and $1.602\,176\,53 \times 10^{-19} \text{ C} \pm 0.000\,000\,14 \times 10^{-19} \text{ C}$ by the Committee on Data for Science and Technology (CODATA). CODATA is an interdisciplinary Scientific Committee, established in 1966 by the International Council for Science (ICSU) to promote and encourage the compilation, evaluation and dissemination of reliable numerical data of importance to science and technology on a world-wide basis.

11. Practical Realization of the ampere (A)

Laboratory reference standards for the volt and the ohm based upon the Josephson and quantum-Hall effects are, however, significantly more reproducible and stable than a few parts in 10^7 . The introduction on 1 January 1990 of conventional values for the Josephson constant K_{J-90} and the von Klitzing constant R_{K-90} , following their adoption by the CIPM in 1988, has resulted in a significant improvement in the world-wide uniformity of electrical measurements. For the BIPM, the formal introduction of these two quantum effects into electrical metrology has changed the way in which international comparisons at the highest level are carried out. Hence, in practice, the ampere may be reproduced to accuracy better than a few parts in 10^7 from the volt and the ohm via Ohm's Law.

12. The Josephson Effect (JE)

The Josephson effect was discovered by Brian Josephson in 1960s. When a thin insulating barrier, about 1 to 2 nm thick, between two superconductors (i.e. a Josephson junction) is cooled in liquid helium to 4.2 K and exposed to electromagnetic radiation of frequency, f , its current-voltage characteristic exhibits steps at highly precise quantized Josephson voltages, V_J . The discrete value of V_J is given by nf/K_J , where n is an integer and K_J is the Josephson constant, which is equal to the invariant quotient of fundamental physical constants $2e/h$.

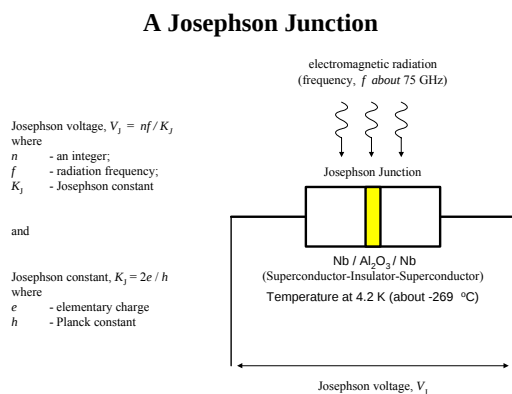


Figure 2: A Josephson Junction

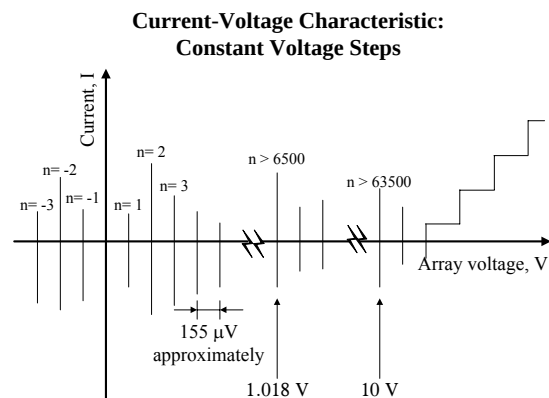


Figure 3: Characteristic Curve for JE

Voltage standards based on arrays of Josephson junctions are commonly used by NMIs. These arrays, which consist of several thousand individual junctions, can be used to measure voltage standards directly at the 1 V and 10 V level. According to BIPM, the reproducibility of a voltage standard using an array of Josephson junctions can achieve a few parts in 10^{10} . No other voltage standard, such as the Zener standard, has stability comparable with this. On 1 January 1990, to take advantage of this highly stable method of maintaining reference voltage standards, the CIPM introduced the conventional value $K_{J-90} = 483597.9$ GHz/V exactly and the uncertainty of the value of K_{J-90} in the SI system is not included in the estimates of measurement uncertainties, for realization the unit of volt.

13. The Quantum Hall Effect (QHE)

The quantum Hall effect was discovered by K. von Klitzing, G. Dorda and M. Pepper in 1980s. They demonstrated that the quantised Hall resistance could be measured in silicon MOSFETs at low temperatures and in high magnetic fields. The quantised Hall effect is most easily observed in gallium arsenide/gallium aluminium arsenide (GaAs/GaAlAs) heterostructure devices.

In these devices, the electrons have very high mobility in a quasi-two-dimensional region of the GaAs close to the GaAs/GaAlAs interface. With current, I , flowing through the two-dimensional layer, the Hall voltage, V_H , and the longitudinal voltage, V_{xx} are measured. The quantised Hall resistance, R_H , and longitudinal resistance, R_{xx} , are given by the ratios V_H to I and V_{xx} to I respectively. At the plateaux of the characteristic curve, R_{xx} is equal to zero and R_H is observed to be constant and independent of magnetic field and device geometry. In fact, at $V_{xx} = 0$, the value of R_H is found to be integer multiples and is given by R_K/i , where i is an integer and R_K is the von Klitzing constant, which is equal to the invariant quotient of fundamental physical constants h/e^2 .

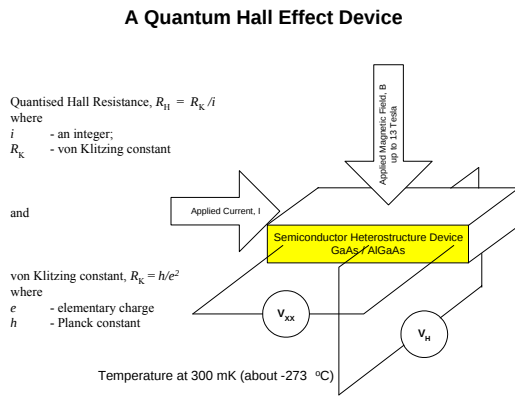


Figure 4: A QHE Device

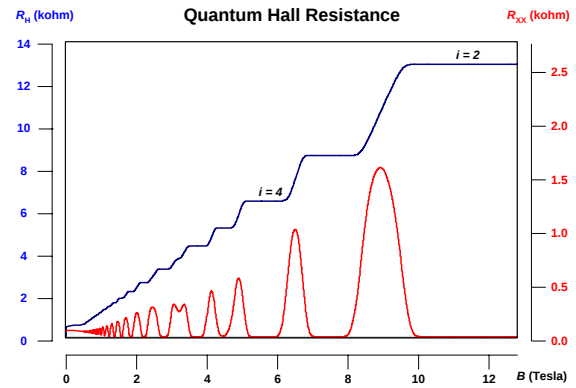


Figure 5: Characteristic Curve for QHE

Many NMIs, including SCL, have been using resistance standards based on the quantum Hall effect. According to BIPM, the reproducibility of quantum Hall effect based resistance standards can be as precise as a few parts in 10^9 . No other resistance standard, such as wire-wound resistors, has stability comparable with this. On 1 January 1990, to take advantage of this method of maintaining reference resistance standards, the CIPM introduced the conventional value $R_{K-90} = 25812.807 \Omega$ exactly and the uncertainty of the value of R_{K-90} in the SI system is not included in the estimates of measurement uncertainties, for realization of the unit of ohm.

14. The Josephson Array Voltage Standard (JAVS) at SCL

The SCL JAVS is based on a 10-V Josephson array chip of 20208 Nb-Al₂O₃-Nb Josephson junctions, made by Hypres Inc, from the original design of the National Institute of Standards and Technology (NIST). The SCL system uses a Millitech Gunn diode operating at about 75 GHz source stabilized by an EIP 578B counter as the microwave frequency source. The microwave frequency source is coupled to the Josephson array via a dielectric waveguide. The 10 MHz time base frequency of the EIP 578B counter is supplied from a HP 5061B Caesium Beam Frequency Standard with accuracy better than 1 part in 10^{12} . Step selection

and bias of the Gunn oscillator is carried out using an Astro Endyne JBS500 Josephson Bias Source. The filters at the output of the JAVS have a series resistance of $20\ \Omega$ and a leakage of $4 \times 10^{10}\ \Omega$.

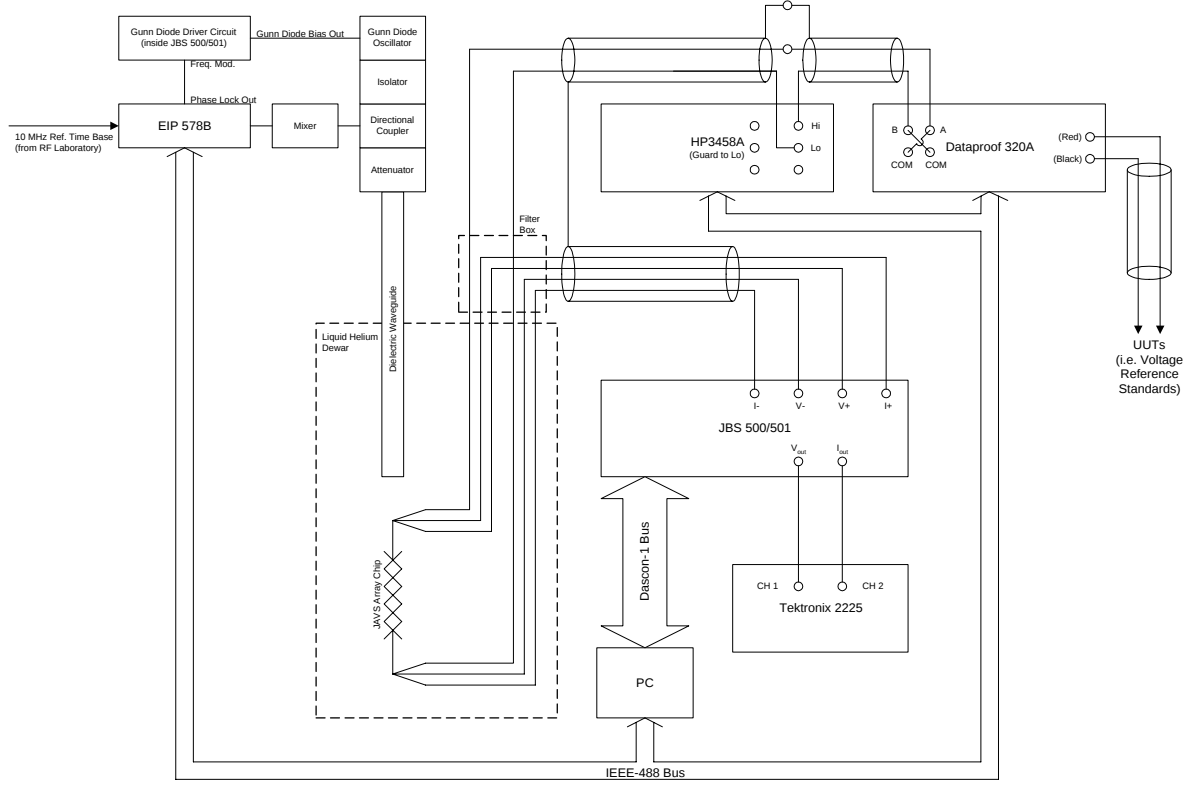


Figure 6: Schematic Diagram of the SCL JAVS System

Electronic DC voltage reference standards (VRS) are calibrated by comparison against the JAVS via an HP 3458A digital multimeter, which is used as a high precision null detector. During the calibration process, either a Dataproof 320A standard cell scanner or a Guildline 9145A low thermal selector switch is used to connect VRS under test to the JAVS. Measurements are made under computer control and the software used is NISTVolt 5.2.

Estimated measurement uncertainties for calibration of VRS against the JAVS:

Uncertainty Source	Type	Prob. Dist.	Standard Uncertainty ($\times 10^{-9}$)		degrees of freedom
			1.018 V	10 V	
Frequency	B	Normal	0.07	0.07	>200
Leakage	B	Normal	0.29	0.29	>200
Offset	B	Normal	40	6	>200
Null Voltage	B	Normal	6	0.6	>200
Noise / Random	A	Normal	20	3	39
Combined Standard Uncertainty			45	6.7	
Expanded Uncertainty (coverage factor, $k=1.96$)			8.8	13	

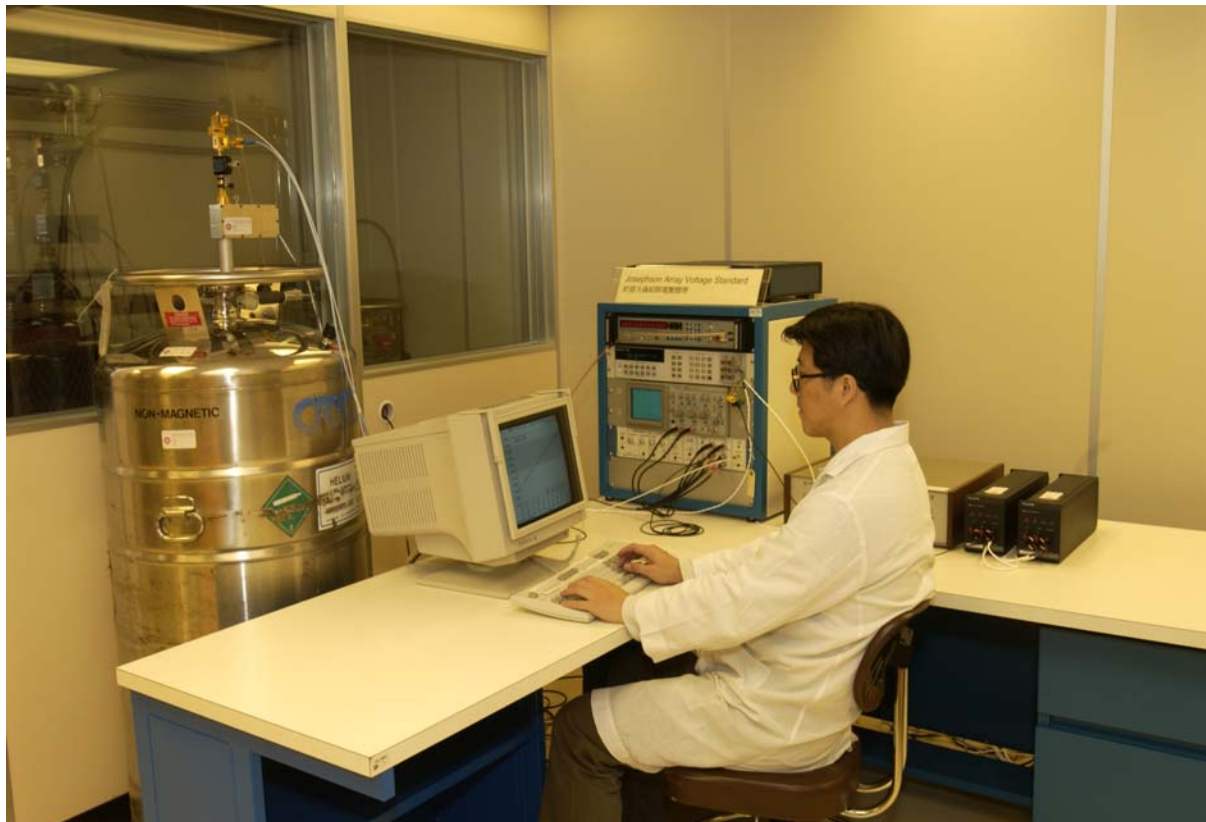


Figure 7: Operation of the SCL JAVS System

15. The Quantised Hall Resistance Standard (QHRS) at SCL

The SCL QHRS system consists of a magnet, a GaAs-GaAsAl heterostructure Quantum Hall Effect (QHE) device, a ^3He variable temperature sample insert and an automatic cryogenic current comparator (CCC) resistance bridge. With the CCC resistance bridge, the Laboratory's very stable $100\ \Omega$ standard resistor is calibrated against the Hall resistances, $R_H(i)$, of the QHE sample. Other in-house resistance standards are calibrated

against this highly stable $100\ \Omega$ standard resistor.

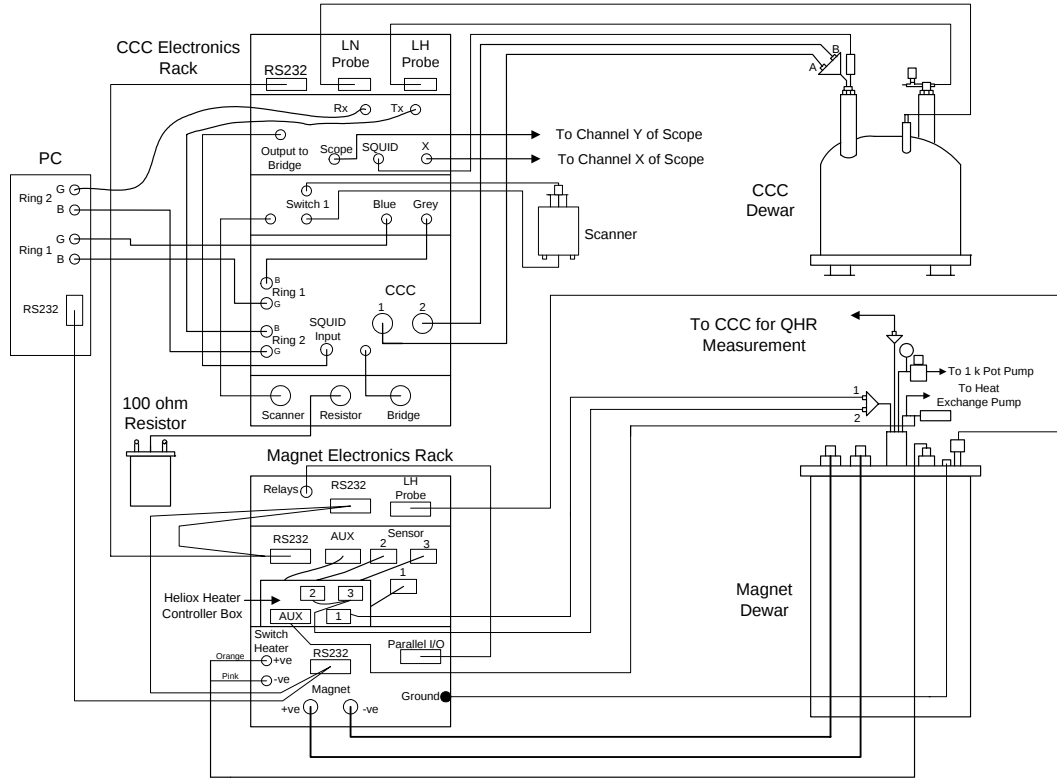
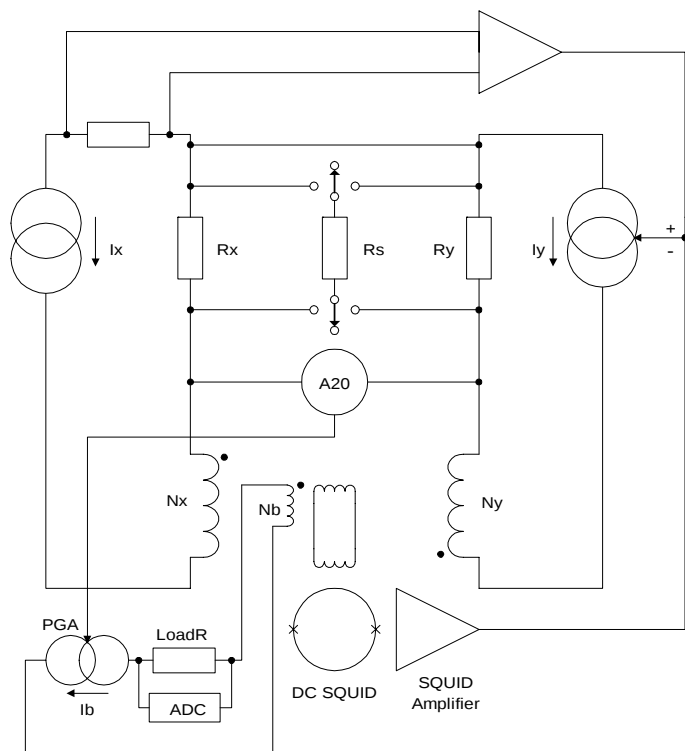


Figure 8: Schematic Diagram of the SCL QHRS System

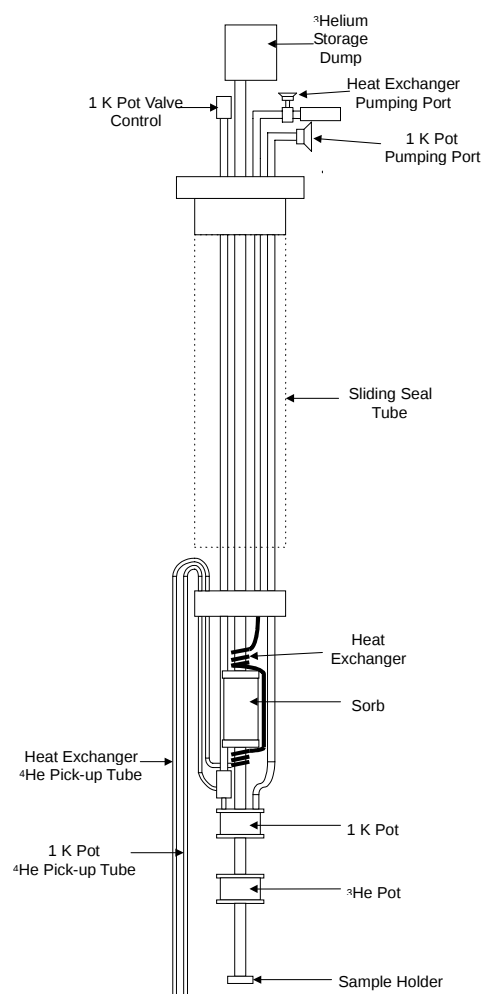
The CCC resistance bridge uses a DC SQUID to measure the magnetic field imbalance in the current comparator toroid in order to determine the resistance ratios of the reference and test resistors. The CCC provides a binary sequence of windings so that its winding errors can be verified to within 1 part in 10^9 using a binary build-up technique. This CCC resistance bridge is capable to perform resistance measurements in the range from $1\ \Omega$ to $10\ \text{k}\Omega$ with resolution of 1 part in 10^9 and combined standard uncertainty of about 5 parts in 10^9 .

The ^3He sample insert provides automated control of the sample temperature that can be varied from 300 mK to 4 K. The sample is installed inside a vacuum chamber with no direct contact with ^3He . The sample insert is located inside the core of the magnet, which is operated at 4.2 K with a maximum flux density of 14 T.

A Tinsley 5685A $100\ \Omega$ standard resistor, housed in a temperature controlled air bath, is used as the system's Resistance Transfer Device (RXD). To minimize measurement uncertainties, this $100\ \Omega$ RXD is first calibrated against the QHR resistance and it will then be used as the reference to calibrate other resistors.



The Automatic CCC Resistance Bridge



The QHE Device Insert

Figure 9: Major Components of the SCL QHRS System

Estimated measurement uncertainties for calibration of standard resistors against the QHRS:

Uncertainty Source	Type	Prob. Dist.	Standard Uncertainty ($\times 10^{-9}$)		degrees of freedom
			RXD against QHR	Resistors against RXD	
CCC Bridge Ratio	B	Normal	1.5	1.5	>100
CCC Null Compensation	B	Normal	1.0	1.0	>100
CCC Leakage	B	Normal	0.1	0.1	>100
CCC Instability	B	Normal	1.0	1.0	>100
QHE Insert Leakage	B	Normal	7.5	---	>100
Noise / Random	A	Normal	5.0	5.0	9
System's RTD	B	Normal	---	9.2	>100
Combined Standard Uncertainty			9.2	10.6	
Expanded Uncertainty (coverage factor, $k=1.96$)			18	21	

Note: RXD stands for System's 100 Ω Resistance Transfer Device



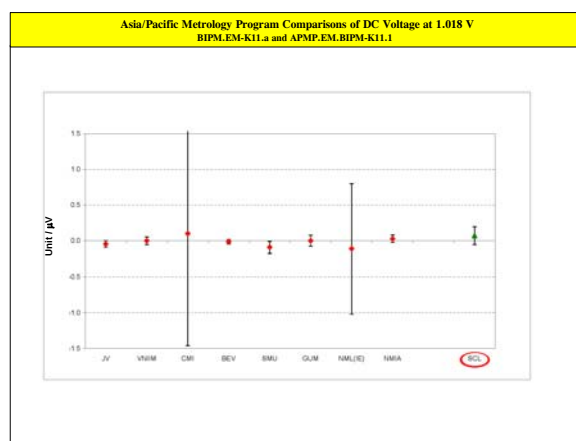
Figure 10: Operation of the SCL QHRS System

16. Participation in International Comparisons by SCL

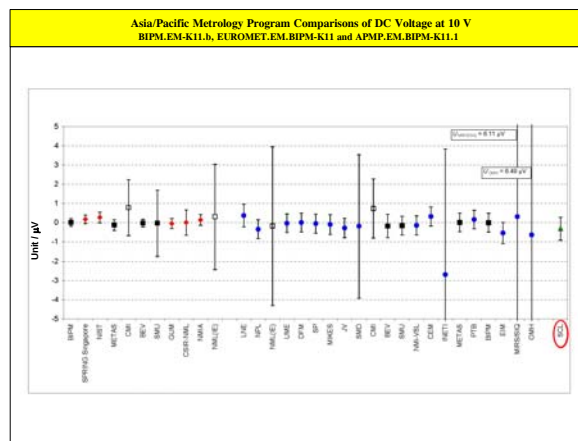
As mentioned above, the CIPM MRA is an arrangement among NMIs for the mutual recognition of national measurement standards and for recognition of the validity of calibration and measurement certificates issued by NMIs. The technical basis for mutual recognition of national measurement standards among NMIs is derived from the results of key comparisons, for the quantities and values specified in Appendix B of the CIPM MRA. This comprises the set of results obtained in the course of time through key comparisons carried out by the Consultative Committees of the CIPM, the BIPM and the regional metrology organizations (RMOs), and published by the BIPM and maintained in the BIPM key comparison database. Hence, it is necessary for SCL to participate in international comparison for relevant measurement quantities.

With the primary standards for electrical measurements established, SCL has participated in relevant inter-laboratory comparisons. In fact, these results have been published in Appendix B of the CIPM MRA, on the BIPM website. The results provide sound

technical basis for the recognition of SCL's electrical primary standards, namely the JAVS and QHRS Systems, by other overseas NMIs.

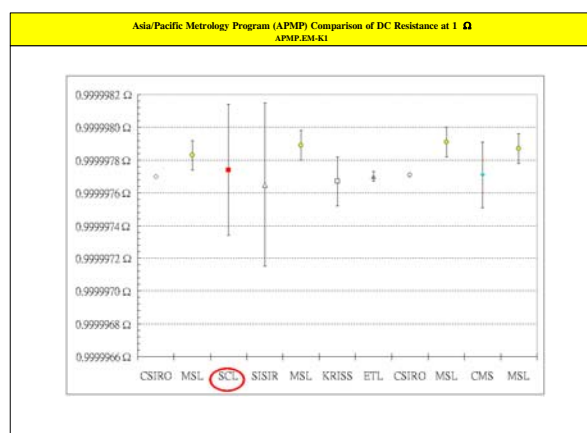


Agreement within 6.9×10^{-8}

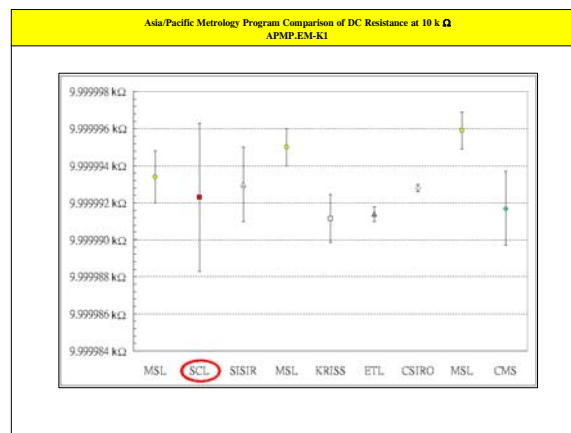


Agreement within 3.3×10^{-8}

Figure 11: International Interlaboratory Comparison Results for DC Voltage



Agreement within 3.2×10^{-8}



Agreement within 6.6×10^{-8}

Figure 12: International Interlaboratory Comparison Results for DC Resistance

References

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